The Pressure Field in an Unsteady-State Fluidized Bed

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An assumption commonly, but by no means universally, made in the stability analysis of the homogeneously fluidized state is that the pressure field may be well approximated by its gravitational component alone. This assumption is critically examined: the results indicate that it is valid for most cases of practical interest but can lead to significant discrepancies for liquid fluidization of low-density particles.

The analysis of fluidized-bed behavior on the basis of formulations of the relevant equations of change has been the subject of sustained study for more than a quarter of a century: a survey of this field is provided by Jackson (1985), and some recent developments and interpretations of the physical mechanisms contributing to the momentum balance equations are discussed by Foscolo and Gibilaro (1987) and Batchelor (1988). A motivation for this work has been to find an answer to the intriguing question of why it is that some fluidized beds expand with increasing fluid velocity in a stable homogeneous manner, while others show heterogeneous bubbling characteristics that give the appearance of a vigorously boiling liquid.

Gas-fluidized fine powders (in the 20 to 200 μm size range) can exhibit both types of behavior with an abrupt transfer from the homogeneous to the bubbling regime occurring at a well defined value of the fluid velocity and void fraction; and certain liquid-fluidized systems, normally associated with stable homogeneous behavior, can undergo similar transformations to a bubbling state and be made to resemble, in almost every respect, a normal gas-fluidized bed. Many data for the minimum bubbling void fraction, ϵ_{mb}, at which this transition occurs, are reported for gas fluidization (see, for example, Gibilaro et al., 1988) and have been correlated by Geldart (1973) for ambient air fluidization; a counterpart correlation for ambient water fluidization is reported by Gibilaro et al. (1986). A major goal for the theoretical studies must be the prediction of these minimum bubbling points and their dependence on the physical properties of the fluidized systems under consideration.

Formulations of the equations of change in general assume incompressible fluid and particle phases and a bed that is statistically homogeneous in any horizontal plane; they are also based on a control volume that, although assumed to contain many particle layers, is nevertheless small in comparison with the wave length of the perturbations to be considered in the subsequent analysis. With these assumptions, and disregarding contributions to the net momentum flux brought about by random velocity fluctuations and added mass terms due to the relative acceleration of the phases, the equations representing conservation of mass and momentum in the fluid and particle phases become:

Conservation of Mass

Fluid phase

$$\partial\epsilon/\partial t + \partial[\epsilon \cdot u_f]/\partial z = 0 \tag{1}$$

Particle phase

$$\partial\epsilon/\partial t - \partial[(1-\epsilon) \cdot u_P]/\partial z = 0 \tag{2}$$

Conservation of Momentum

Fluid phase

$$\rho_f \cdot \epsilon \cdot Du_f/Dt = -F_{Gf} - F_I - \partial p_f/\partial z \tag{3}$$

Particle phase

$$\rho_P \cdot (1-\epsilon) \cdot Du_P/Dt = -F_{GP} + F_I - \partial p_P/\partial z \tag{4}$$

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The three terms on the righthand side of each of the two momentum equations represent the forces imparted to the phase in question in the control volume by the action of gravity, by interaction with the other phase in the control volume, and across the horizontal surface of the control volume, respectively. Of these six forces it is the "particle pressure gradient," $\partial p_P/\partial_z$, that occupies the limelight in speculative considerations: it is that term that provides an effective elasticity for the particle phase, thereby allowing for the possibility of stable homogeneous fluidization. The physical mechanisms responsible for "particle pressure" remain, however, a subject for intense debate that will not be considered further here.

Rather, it is the purpose of this paper to consider critically the assumption that accelerational components of the pressure field can be disregarded; this assumption is examined in the light of a formulation that provides constitutive expressions for the particle pressure gradient and fluid-particle interaction force, thereby closing the above equations. The conclusion is that the approximation is valid for the analysis of the instability of the uniformly expanded state for most cases of practical interest.

The Particle Bed Model

Rather than treat the particle phase as a pseudo-fluid continuum, the particle bed model (Foscolo and Gibilaro, 1987) employs the idealization of regular "expanded close packed" particle layers: a layer contains N_L particles per unit area of bed cross section,

$$N_L = 4 \cdot (1 - \epsilon)/(\pi \cdot d_P^2), \tag{5}$$

and there are Nv particles per unit volume,

$$N_v = 6 \cdot (1 - \epsilon)/(\pi \cdot d_P^3). \tag{6}$$

The fluid-particle interaction force, F_I, which acts on all particles with their centers within the control volume, is considered as the sum of long- and short-range effects. Thus,

$$F_I = F_B + F_D, \tag{7}$$

where F_B is the effect of the mean macroscopic pressure field,

$$F_B = -N_v \cdot (\pi \cdot d_P^3/6) \cdot \partial p_f/\partial z, \tag{8}$$

and F_D is due to local distortions of the flow field around individual particles, and is obtained in terms of the equilibrium bed expansion parameters, u_t and n, for which correlations are readily available:

$$F_D = N_v \cdot (\pi \cdot d_P^3/6) \cdot (\rho_P - \rho_f) \cdot g \cdot [(u_o - u_P)/u_t]^{4.8/n} \cdot \epsilon^{-3.8} \tag{9}$$

This expression for F_D does not contain the fluid interstitial velocity, u_f; this is because the formulation is in terms of a constant volumetric flux, u_o, and of incompressible phases, so that the fluid-particle slip velocity may be expressed independently of u_f.

The "particle pressure" gradient may be expressed in terms of the effective elastic modulus of the particle phase, E_P:

$$\partial p_P/\partial z = -E_P \cdot \partial\epsilon/\partial z. \tag{10}$$

This represents a force that acts solely on the particle layers at the boundary of the control volume under nonequilibrium conditions. In the particle bed model it is attributed to the dependence of the fluid-particle interaction force on particle concentration, which yields the constitutive relationship:

$$E_P = N_L \cdot 4.8 \cdot \pi \cdot d_P^3/6 \cdot (\rho_P - \rho_f) \cdot g. \tag{11}$$

The expressions listed above, Eqs. 5–11, close the equations of change, Eqs. 1–4. By now considering the case of small perturbations of voidage imposed on a previously homogeneous steady-state bed, its stability can be examined: voidage disturbances that increase with time may be associated with the bubbling regime, and those that decay, with the stable homogeneous state.

Pressure field approximation: single-phase analogy

In the previous examination of the stability of the state of homogeneous fluidization on the basis of the particle bed model (Foscolo and Gibilaro, 1987), the full description outlined above was not employed. Instead, the assumption was made that the fluid pressure gradient could be approximated by the expression for its mean value in a homogeneous unperturbed bed, namely:

$$\partial p_f/\partial z = -[(1 - \epsilon) \cdot \rho_P + \epsilon \cdot \rho_f] \cdot g. \tag{12}$$

As the stability analysis is in terms of small perturbations about the unperturbed state, this appeared a reasonable assumption. Its adoption has the effect of decoupling the fluid and particle momentum equations, with the result that the particle phase equations take a form that is essentially analogous to that of a single compressible fluid.

On this basis, the dynamic wave velocity of this pseudo-single-phase system has been obtained and may be expressed,

$$u_{e1} = \sqrt{(E_P/\rho_P)} = \sqrt{[3.2 \cdot g \cdot d_P \cdot (1 - \epsilon) \cdot (\rho_P - \rho_f)/\rho_P]}, \tag{13}$$

which, together with the well established expression for the continuity wave velocity, u_ϵ,

$$u_\epsilon = u_t \cdot n \cdot (1 - \epsilon) \cdot \epsilon^{(n-1)}, \tag{14}$$

provides an explicit expression for Wallis's criterion for instability, namely:

$$\sqrt{(g \cdot d_P)}/u_t \cdot \sqrt{[(\rho_P - \rho_f)/\rho_P]} - 0.56 \cdot n \cdot \sqrt{(1 - \epsilon_0)} \cdot \epsilon_0^{n-1} = \begin{cases} +\text{ve, stable} \\ 0\text{, stability limit} \\ -\text{ve, unstable} \end{cases} \tag{15}$$

The expression on the lefthand side of Eq. 15 is in terms of the fundamental fluid and particle properties and the equilibrium bed expansion parameters, u_t and n, that follow from these properties. It therefore provides a quite specific answer to the question posed in the opening paragraph of this paper, and has stood up well to the test of confrontation with the experimental evidence: Gibilaro et al., (1988) show that the reported effect on the minimum bubbling voidage for gas-fluidized beds of systematically varying parameters in Eq. 15, is in good agreement with predictions, as are the less abundant data on liquid-fluidized systems (Gibilaro et al., 1986).

The two-phase analysis

Notwithstanding the success of Eq. 15 in accounting for the observed behavior, questions regarding its formulation remain that are not, in general, easily resolved. Batchelor (1988), for example, arrives at an instability criterion, through consideration of other hydrodynamic mechanisms, that appears to bear some numerical similarity to Eq. 15 for the case of gas fluidization: unfortunately a more precise evaluation is not possible as it contains a parameter for which correlations in terms of known system properties are as yet unavailable.

The question of the pressure field approximation, however, may be readily resolved following the general procedure of Wallis (1969) for two-phase flow: this is the procedure adopted for gas-fluidized beds by Fanucci et al. (1979). It simply involves eliminating the unknown fluid pressure gradient term by combining the fluid and the particle momentum equations. $\partial p_f/\partial z$ appears explicitly in Eq. 3 and is also included in the fluid-particle interaction term, F_I (Eqs. 7 and 8), in both Eqs. 3 and 4. Performing this elimination we have:

$$\rho_f \cdot Du_f/Dt - \rho_P \cdot Du_P/Dt = (\rho_P - \rho_f) \cdot g - F_D/[(1 - \epsilon) \cdot \epsilon] - E_P/(1 - \epsilon) \cdot \partial\epsilon/\partial z, \quad (16)$$

Eq. 16 replaces the particle-phase momentum equation, Eq. 4, of the single-phase treatment outlined above. The main difference lies in the fluid accelerational contribution whose importance clearly diminishes for high values of particle density relative to the fluid density.

Linearization of Eqs. 1, 2, and 16, about the unperturbed, steady-state condition ($\epsilon = \epsilon_o$; $u_f = u_{fo} = u_o/\epsilon_o = u_t \cdot \epsilon_o^{(n-1)}$; $u_P = 0$), and subsequent manipulation to eliminate the fluid and particle velocity variables, yields the second-order equation for voidage perturbations, ϵ', in a fluidized bed:

$$\partial^2\epsilon'/\partial t^2 + 2 \cdot v_o \cdot \partial^2\epsilon'/\partial z\partial t + A \cdot \partial^2\epsilon'/\partial z^2 + B \cdot \rho_P/[(1 - \epsilon_o) \cdot \rho_f + \epsilon_o \cdot \rho_P] \cdot (\partial\epsilon'/\partial t + u_\epsilon \cdot \partial\epsilon'/\partial z) = 0; \quad (17)$$

where:

$$A = [(1 - \epsilon_o) \cdot \rho_f \cdot u_{fo}^2 - \epsilon_o \cdot E_P]/[(1 - \epsilon_o) \cdot \rho_f + \epsilon_o \cdot \rho_P]; \quad (18)$$

$$B = 4.8 \cdot g \cdot (\rho_P - \rho_f)/(n \cdot u_t \cdot \rho_P \cdot \epsilon_o^{n-1}); \quad (19)$$

$$v_o = (1 - \epsilon_o) \cdot \rho_f \cdot u_{fo}/[(1 - \epsilon_o) \cdot \rho_f + \epsilon_o \cdot \rho_P]. \quad (20)$$

Eq. 17 is formally identical to that obtained by Wallis (1969) in his generalized treatment of two-phase flow. The parameter, v_o, represents his "weighted mean velocity" relative to which dynamic- and continuity-wave velocities are conveniently expressed:

$$v_e = u_{e2} - v_o \quad (21)$$

$$v_\epsilon = u_\epsilon - v_o, \quad (22)$$

where u_{e2} is the dynamic wave velocity relative to the mean particle velocity ($u_{Po} = 0$) in the full, two-phase treatment.

Wallis (1969) has demonstrated that

$$v_e^2 = v_o^2 - A, \quad (23)$$

and that the two-phase criterion for instability takes a form analogous to the single-phase formulation, namely

$$v_\epsilon^2 > v_e^2 \quad (24)$$

Thus the boundary between regions of stable and unstable behavior is defined, as before, by the equality of dynamic- and continuity-wave velocities.

On the basis of the explicit relationships reported above, the two-phase stability criterion takes the form:

$$\sqrt{(g \cdot d_P)}/u_t \cdot \sqrt{[(\rho_P - \rho_f)/\rho_P]} - 0.56 \cdot n \cdot \sqrt{(1 - \epsilon_0)} \cdot \epsilon_0^{n-1} \cdot \alpha = \begin{cases} +\text{ve, stable} \\ 0\text{, stability limit} \\ -\text{ve, unstable} \end{cases} \quad (25)$$

where

$$\alpha = \sqrt{(1 + \rho_f/\rho_P \cdot [1 - n \cdot (1 - \epsilon_o)]^2/[n^2 \cdot \epsilon_o \cdot (1 - \epsilon_o)])}. \quad (26)$$

Eq. 25 differs from its single-phase counterpart, Eq. 15, solely in the inclusion of the dimensionless parameter α.

Discussion

The effect of the assumption of a pressure field entirely determined by its gravitational component, Eq. 12, can now be evaluated with regard to the two criteria for instability, Eqs. 15 and 25, and the respective expressions for the velocity of a dynamic wave.

The stability criteria

The expression for α, Eq. 26, shows it to be a positive quantity always greater than unity; this means that the single-phase approximation can predict a somewhat greater voidage range of stable operation for a particular system than the more complete formulation of Eq. 25. On the other hand, it is immediately apparent that α approaches unity as the ratio of particle to fluid density becomes large—as is clearly the case for all gas-fluidized beds. For the case of liquid fluidization, however, significant deviations in the predictions of the two criteria would appear possible.

These observations are borne out by the examples illustrated in Figures 1 and 2. These show values of α as functions of the full operating range of void fraction for a typical gas-fluidized fine powder and liquid-fluidized glass spheres, respectively. It should be pointed out at this stage that although α approaches infinity as the void fraction approaches unity, the second term in the criterion, Eq. 25, is bounded and assumes a value of $0.56 \cdot \sqrt{(\rho_f/\rho_P)}$ at $\epsilon_o = 1$; for the single-phase criterion, Eq. 15, this limit is zero.

For the gas-fluidization case, the deviation of α from unity is clearly negligible over the whole range of operation. For liquid fluidization, on the other hand, α shows a clear minimum at a

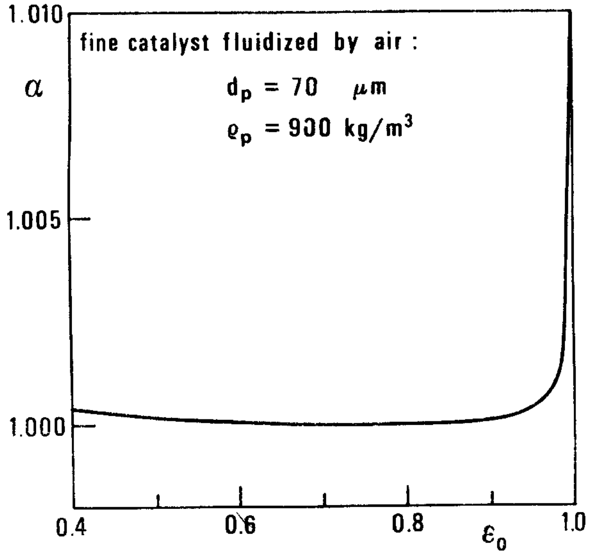

Figure 1. ***α*** **vs. void fraction for a typical fine catalyst fluidized by ambient air.**

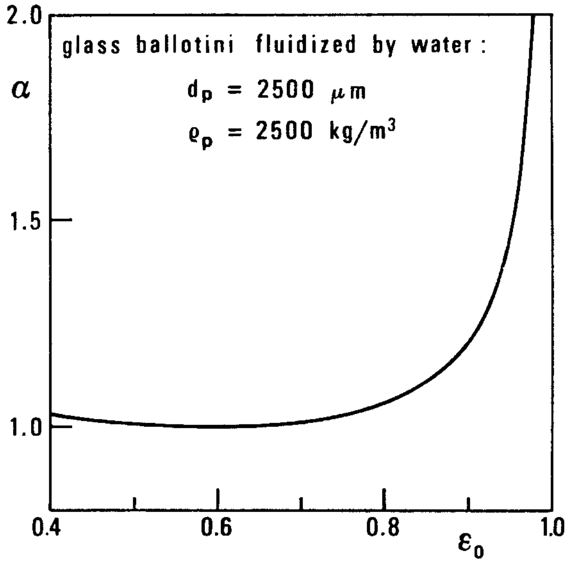

Figure 2. ***α*** **vs. void fraction for glass ballotini fluidized by water.**

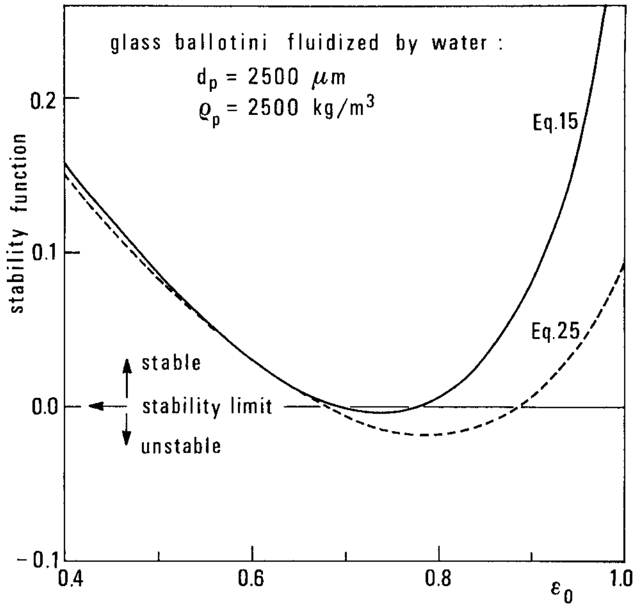

Figure 3. Single-phase vs. two-phase stability criteria for glass ballotini fluidized by water.

void fraction of about 0.6, but does not assume a significant value as far as its contribution to the stability prediction is concerned until high void fraction values are reached. This latter point is well illustrated in Figure 3, which shows values of the "stability function" (i.e., the expressions on the lefthand sides of Eqs. 15 and 25) for the liquid-fluidization example of Figure 2, as functions of voidage: for void fractions less than about 0.7 there is no significant deviation in the two functions; thereafter, deviations become progressively more pronounced, with the two-phase criterion, Eq. 25, predicting a significant region of unstable behavior.

Some further numerical predictions are presented in Table 1. This shows evaluations of both the minimum bubbling voidage, ϵ_{mb}, and the voidage, ϵ_{df}, at which unstable, bubbling, systems show a return to homogeneous behavior (Figure 3 illustrates these two solutions to the criteria equations for a specific example). It is clear that for air, even at high pressure, there is nothing to choose between the single- and the two-phase formulations. The same is true for water fluidization of high-density solids, but for glass, the second transition point is significantly affected; the close proximity of predictions for the first transition, however, is reassuring, given the reasonable agreement reported for the onset of instability between predictions of the

Table 1. Transition to Bubbling and to Dilute Fluidization Predicted by Single-Phase and Two-Phase Criteria for Typical Gas- and Liquid-Fluidized Systems

	Particles			ϵ_{mb}		ϵ_{df}	
Fluidized by	Material	ρ_P kg/m³	d_P μm	Eq. 15	Eq. 25	Eq. 15	Eq. 25
Ambient air	Glass	2,500	50	0.490	0.490	0.997	0.997
" "	Alumina	900	70	0.579	0.579	0.992	0.992
at 5 MPa	Alumina	900	70	0.706	0.707	0.928	0.933
Ambient Water	Copper	8,700	400	0.574	0.574	0.943	0.956
" "	Glass	2,500	2,500	0.694	0.679	0.777	0.887

single-phase criterion and experimental observations (Gibilaro et al., 1986).

Velocities of dynamic waves

The feature of the particle bed model formulation that most distinguishes it from alternative treatments of fluidization dynamics lies in the expressions that emerge for the dynamic wave velocities solely in terms of the basic system properties: it is this that enables precise predictions to be made concerning the stability of the uniform fluidized state.

For the single-phase approximation, the dynamic wave velocity, u_{e1}, is given by Eq. 13. For the two-phase formulation, it can be evaluated from the expressions for A and v_o (Eqs. 18 and 20) and may be written:

$$u_{e2} = (R \cdot u_{fo} + \sqrt{[u_{e1}^2 + R \cdot (u_{e1}^2 - u_{fo}^2)]})/(1 + R), \qquad (27)$$

where

$$R = (1 - \epsilon_o) \cdot \rho_f/(\epsilon_o \cdot \rho_P). \qquad (28)$$

When the density ratio, ρ_f/ρ_P, is small, as is certainly the case for all gas-fluidized beds, R approaches zero and u_{e2} approaches the single-phase expression, u_{e1}.

For liquid fluidization it is not possible to make such a generalization, and individual cases must be examined independently. For ambient water fluidization, deviations of u_{e2} from u_{e1} increase with both decreasing particle density and decreasing particle size.

Although it is not, in general, possible to measure dynamic wave velocities directly, Wallis (1962) describes a method for estimating them for the case of incipient fluidization, $\epsilon_o = \epsilon_{mf} = 0.4$; this involves measurement of the velocity of the interface between a section of bed held against a mesh at the top of the column and the liquid below, into which the lower particles can be made to rain.

Extrapolating this "dynamic shock" velocity to its value at a fluid velocity equal to the minimum fluidization velocity, u_{mf}, yields the dynamic wave velocity for $\epsilon_o = \epsilon_{mf}$. This procedure has been applied to liquid fluidization of a range of particle species by Hossain (1987), and Figure 4 shows typical measured dynamic shock velocities for four different systems, together with the predictions of Eqs. 13 and 27, to which these shocks should extrapolate at $u_o/u_{mf} = 1$.

These results bear out the conclusions reported by Gibilaro et al. (1989) that, for the low-density plastic particles, u_{e1} significantly overestimates the measured dynamic wave velocity, whereas, for the more dense particles, the agreement is satisfactory. u_{e2} provides a better estimate for the low-density particle case but is still too high. This mismatch is not surprising, as added mass effects are likely to become significant as particle density decreases without being incorporated in the model formulation.

For particles of the density of glass and above, the discrepancy between the predictions of the two expressions becomes progressively less significant.

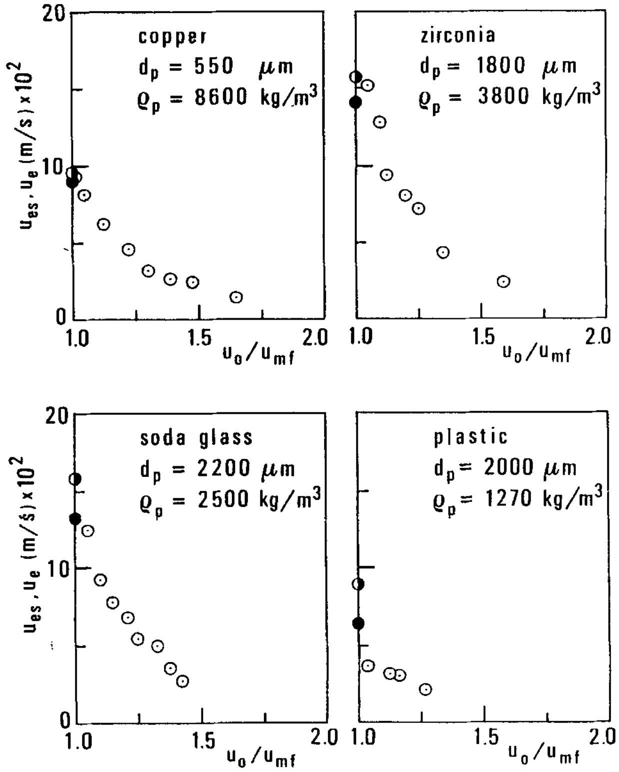

Figure 4. Experimental dynamic shocks and calculated dynamic wave velocities for different water-fluidized systems.

◐: Calculated with Eq. 13; ●: calculated with Eq. 27

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Notation

- A = defined in Eq. 18, m^2/s^2
- B = defined in Eq. 19, 1/s
- d_P = particle diameter, m
- D = substantial derivative
- E_P = elastic modulus of the particle phase, N/m^2
- F_B = action of fluid pressure field on particle phase, N/m^3
- F_D = action of local flow field on particle phase, N/m^3
- F_{Gf} = action of gravity on fluid phase, N/m^3
- F_{GP} = action of gravity on particle phase, N/m_3
- F_I = interaction force, N/m^3
- g = gravitational field strength, N/kg
- n = steady-state expansion parameter
- N_L = number of particles in a horizontal layer per unit area, $1/m^2$
- N_V = number of particles per unit bed volume, $1/m^3$
- p_f = fluid phase pressure, N/m^2
- p_P = particle phase pressure, N/m^2
- R = defined in Eq. 28
- u_e = dynamic wave velocity, m/s
- u_{es} = dynamic shocks velocity, m/s
- u_ϵ = continuity wave velocity, m/s
- u_f = fluid interstitial velocity, m/s
- u_{fo} = steady-state fluid interstitial velocity, m/s
- u_P = particle velocity, m/s
- u_t = particle free-falling velocity, m/s
- u_o = superficial velocity of fluid approaching bed, m/s
- v_e = dynamic wave velocity relative to v_o, m/s
- v_ϵ = continuity wave velocity relative to v_o, *m/s*
- v_o = mean weighted velocity, m/s
- t = time, s
- z = distance, m

Greek letters

α = defined in Eq. 26
∂ = partial derivative
ϵ = bed voidage
ϵ' = voidage perturbation
ϵ_{df} = dilute fluidization transition voidage
ϵ_{mb} = minimum bubbling voidage
ϵ_{o} = steady-state bed voidage
ρ_f = fluid density, kg/m^3
ρ_p = particle density, kg/m^3

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